Inverse Problems in Partial Differential Equations

Inverse problems appear in a broad class of applications where the aim is to deduce unknown information (typically related to physical properties) from observed data. The behavior of the underlying physical system is often governed by partial differential equations (PDEs). Solving the PDEs allows to simulate the system numerically (in the sense of a digital twin). Hence, a natural approach for solving inverse problems is to fit the unknown parameters (e.g., the coefficients of a PDE) so that the system's simulated behavior complies with the real observations.

The practical solution of such inverse coefficient problems highly suffers from ill-posedness and non-linearity effects. In the first part of this talk, we will give a general introduction to inverse problems, demonstrate the problem of ill-posedness, and explain the general idea of overcoming ill-posedness by regularization.

We will then turn to the more specific problem of reconstructing the coefficient of an elliptic PDE from knowledge of the boundary values of its solutions (the famous Calderón problem). We will present recent results on how to overcome the problem of non-linearity by utilizing a special monotonicity and convexity structure of such problems with respect to the semi-definite (Loewner) order. We also derive a new connection to semidefinite convex optimization that allows globally convergent reconstruction algorithms.